# Fresnel Equations Measurements and Analysis for Physics Advanced Lab

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# Index

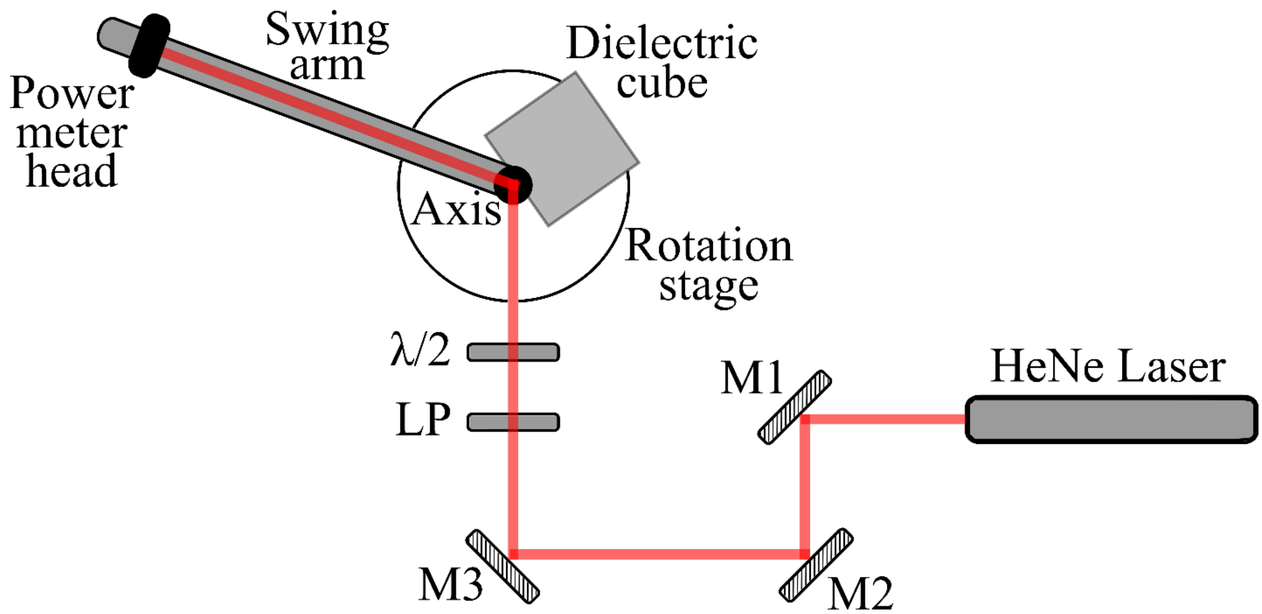

Page 1: Title page
Page 2: Index
Page 3: Workshop description
Page 4: Overview of full Advanced Lab Polarization Unit
Page 5: Schematic of the apparatus used for the Fresnel Equations exercises with potential/suggested sources for equipment
Pages 6-8: Pre-lab readings for students
Page 9: Student tasks for the Fresnel Equations exercises
Page 10: Sample results from the Fresnel Equations exercises (Origin)
Page 11: Screenshot of Excel Solver method of non-linear fit to sample results (Excel)

Workshop Title: The Fresnel Equations; Measurements and Analysis

A relatively simple apparatus and procedure is used to perform measurements of the reflectance from a dielectric surface, which are then analyzed using the Fresnel equations as a model. Our Fresnel equations exercise is one part of a laboratory unit on polarization of optical electromagnetic radiation. We have found this Fresnel equations exercise, though relatively basic, to be consistently and particularly compelling and meaningful for students in probing and cementing their conceptual understanding of polarization in the laboratory context, and in introducing some methods of data analysis.

Outline of Full Adv Lab (PHYS 333) Polarization Unit

1. Malus' Law investigation, demonstration, and measurement
2. Half-wave plate investigation, demonstration, and measurement
3. Birefringence of optical calcite investigation, demonstration, and measurement
4. Fresnel equations investigation, demonstration, and measurement
5. Faraday rotation
    a. investigation, demonstration, and measurement
    b. Verdet constant determination
        i. "simple" approaches
        ii. Modulation spectroscopy approach




1. Mx: mirrors on kinematic mounts; LP: linear polarizer; λ/2: half-wave plate in a rotation mount
2. HeNe laser is unpolarized with stable power output
3. The setup is one portion of an apparatus used for additional exercises in polarization; M3 is positioned as shown to direct the laser to components for the Fresnel equations portion of the exercise and removed to direct the laser to other components for the exercise
4. The ***face*** of the dielectric cube (***not*** its center) is positioned at the rotation axis of the rotation stage; the rotation stage has an angle scale incorporated; the laser beam is aligned to pass through the rotation axis of the rotation stage; the swing arm freely pivots about the same axis as the rotation stage

Potential/Suggested sources for equipment:

- Mirrors, polarizer, λ/2 plate, rotation mount and stage, iris (alignment tool), optomechanics (breadboard, posts, postholders, post bases, kinematic mounts, fixed mounts, clamp (for cube), laser safety screens, hardware, tools, research grade laser and laser power meter): **Thorlabs**
- Acrylic cube: **Eisco**
- Less expensive option for laser and laser power meter: **Industrial Fiber Optics (Educational Products)**
- Swing arm: **fabricated in-house** (no known source)

# Polarization

Polarization is one of the degrees of freedom* of electromagnetic radiation (including, of course, light). For a classical electromagnetic wave, polarization can be thought of as specifying the orientation of the vector electromagnetic fields associated with the wave, though we should remain aware this does not represent a complete understanding of polarization for quantum electromagnetic radiation.

The orientation of the electromagnetic fields in a classical electromagnetic wave varies in both time and position, in a way that can be difficult to visualize, yet this variation can be completely characterized for the wave as a whole in a relatively simple way – its polarization.

We will describe three important manifestations of polarization for classical plane electromagnetic waves. In each case, we will describe what the polarization means in terms of the electric field of the wave at a single point in space over time (the "history" picture of the wave):

- Elliptical polarization: the electric field vector is perpendicular to the direction of propagation of the wave and traces out an ellipse in time
- Circular polarization: the electric field vector is perpendicular to the direction of propagation of the wave and traces out a circle in time
- Linear polarization: the electric field vector remains oriented along a single direction that is perpendicular to the direction of propagation of the wave and varies sinusoidally in time along that direction

Figure 1 is a representation of elliptical polarization for a plane electromagnetic wave that is propagating out of the page toward you (note that the direction of the electric field is perpendicular to the direction of propagation of the wave). It shows the electric field vector at a location *P* at a single instant in time. As time evolves, the magnitude and direction of the electric field vector at *P* would change, rotating continuously clockwise† as represented by the dotted ellipse and the dashed rotation indicator.

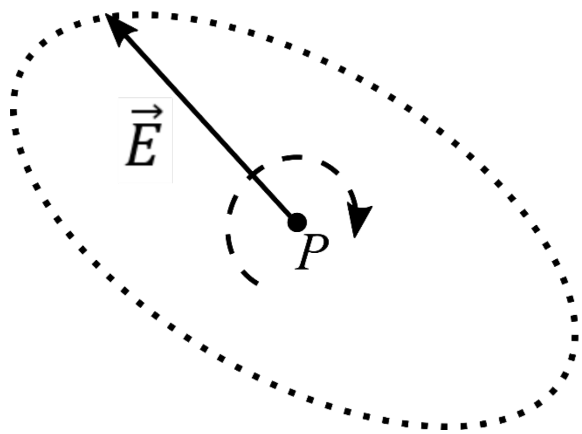



Figure 1: A representation of elliptical polarization. Elliptical polarization is just the general case of polarization for a plane electromagnetic wave, as circular polarization and linear polarization can be seen to be special cases of elliptical polarization.

Considering the elliptical polarization represented in Figure 1, both circular polarization and linear polarization can be seen to be special cases of elliptical polarization: circular polarization is just the special case in which the long and short axes of the ellipse have the same value, i.e., the ellipse is a circle; and linear polarization is just the special case in which the width of the ellipse approaches 0 while its length remains finite.

Finally, in considering Figure 1, keep in mind it shows only the electric field vector (classically there is also a magnetic field associated with an electro***magnetic*** wave) at a single location in space. Different locations also

*A degree of freedom is a parameter of a system whose value affects its interactions independent of all the other parameters describing the system. In the case of polarization, for instance, two photons, quanta of electromagnetic radiation, that are identical in every way ***except*** for polarization, would interact differently with some objects.

† Clockwise rotation was arbitrarily chosen for this diagram. Counterclockwise rotation is also possible. In fact, whether the rotation is clockwise or counterclockwise is part of specifying the polarization state of the electromagnetic radiation, more typically referred to as left-hand or right-hand polarization. Left-hand and right-hand polarization electromagnetic radiation will, generally, interact ***differently*** with matter, so it ***is*** physically significant.

simultaneously experience the electromagnetic fields of the wave, generally at different phases of the elliptical rotation depicted in Figure 1 depending on their position relative to location $P$.

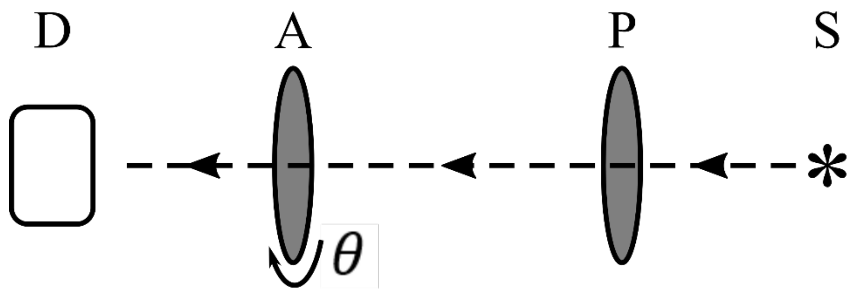



Figure 2: A schematic representation of a basic experimental apparatus to examine aspects of polarization. S: light source; P: polarizer; A: analyzer; D: detector.

The interactions of light with matter are affected by its polarization. Figure 2 is a schematic representation of a basic experimental apparatus that can be used to examine aspects of polarization and its interaction with matter. Light from the source **S** passes through an optical component designated as polarizer, **P**, then through an optical component designated as analyzer, **A**, whose orientation can be varied through angle $\theta$. The irradiance of the light passing through the apparatus, $I$, is measured with detector **D**. As the orientation of the analyzer, $\theta$, is varied, the detected signal for this apparatus follows Malus' law,
$I = I_0 \cos^2 \theta$ . This relationship can be understood as arising through the polarization phenomenon as: the light passing through the polarizer is linearly polarized; the interaction of linearly polarized light with the analyzer will pass only the component of the electric field vector that is parallel to a fixed axis in the material of the analyzer; and the irradiance of electromagnetic radiation is proportional to the absolute-magnitude-squared of the amplitude of its electric field.

As noted several times already, the interaction of light with matter depends on the polarization degree of freedom of the light. ***Birefringent*** materials are identified as those that exhibit two different indices of refraction for different orientations of incident linearly polarized light. Optical components made of birefringent materials – for instance quarter-wave plates and half-wave plates – are used to manipulate the polarization of an incident light beam for various important optical applications. They can be understood as accomplishing this by introducing a relative phase delay of one component of the electric field of the light relative to the perpendicular component. Figure 3 shows a representation of how the relative phase between perpendicular components of the electric field is associated with different polarization states of the electromagnetic wave.

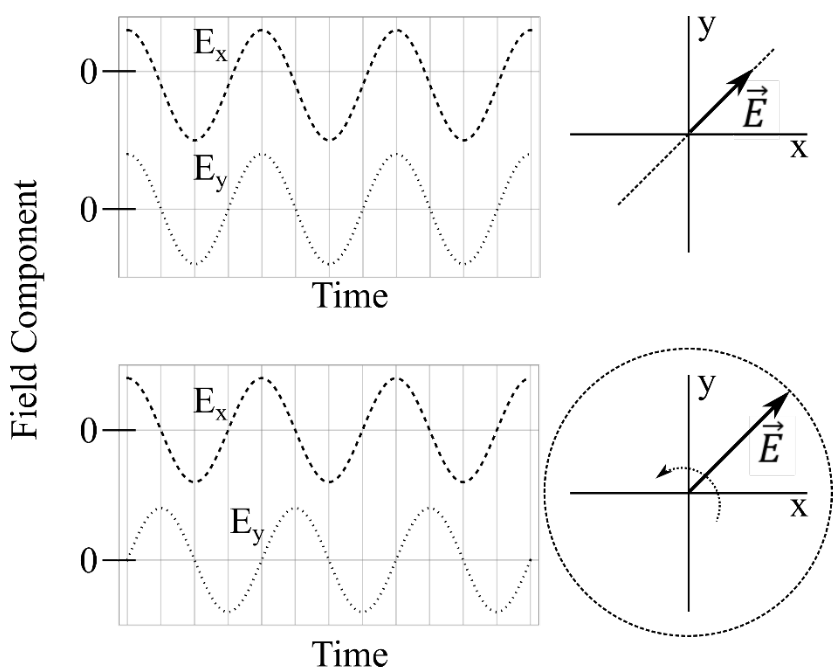



Figure 3: A representation of how relative phase of components of the electric field vector of an electromagnetic wave relate to two possible polarization states: top, linearly polarized, components of equal magnitude and perfectly in phase; bottom, circularly polarized, components of equal magnitude and $\pi/2$ out of phase. That is, introducing a phase delay of $\pi/2$ for one of the components of the electric field for a linearly polarized wave converts it into a circularly polarized wave. In both cases, the direction of propagation of the electromagnetic wave is the z direction. The behavior of the $\vec{E}$ vectors is being considered in a history representation at a single location.

# Dielectric Boundary Geometry

When a plane electromagnetic wave interacts at a dielectric boundary, the relevant geometry for expressing the physics of the interaction is intrinsically three-dimensional and takes some effort to picture and to apply. Figure 1 is a represention of the geometry and essential elements of identified below.

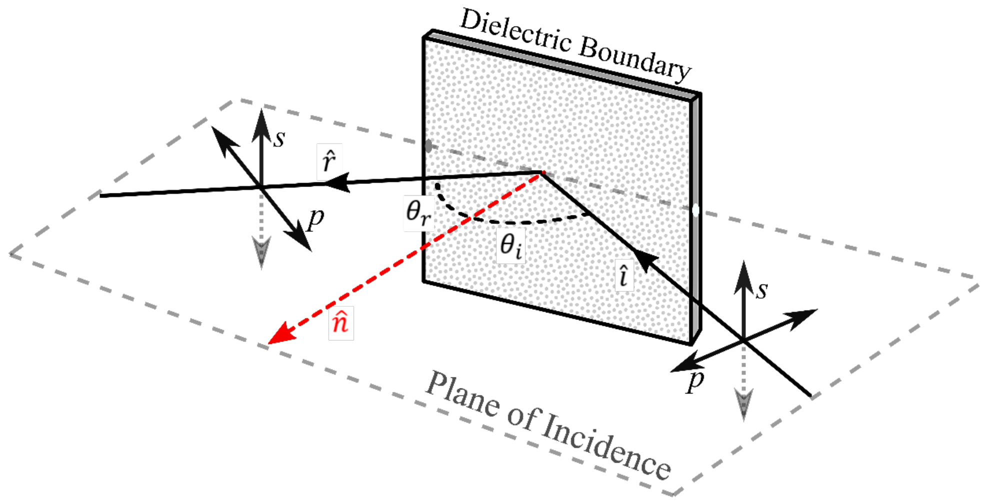


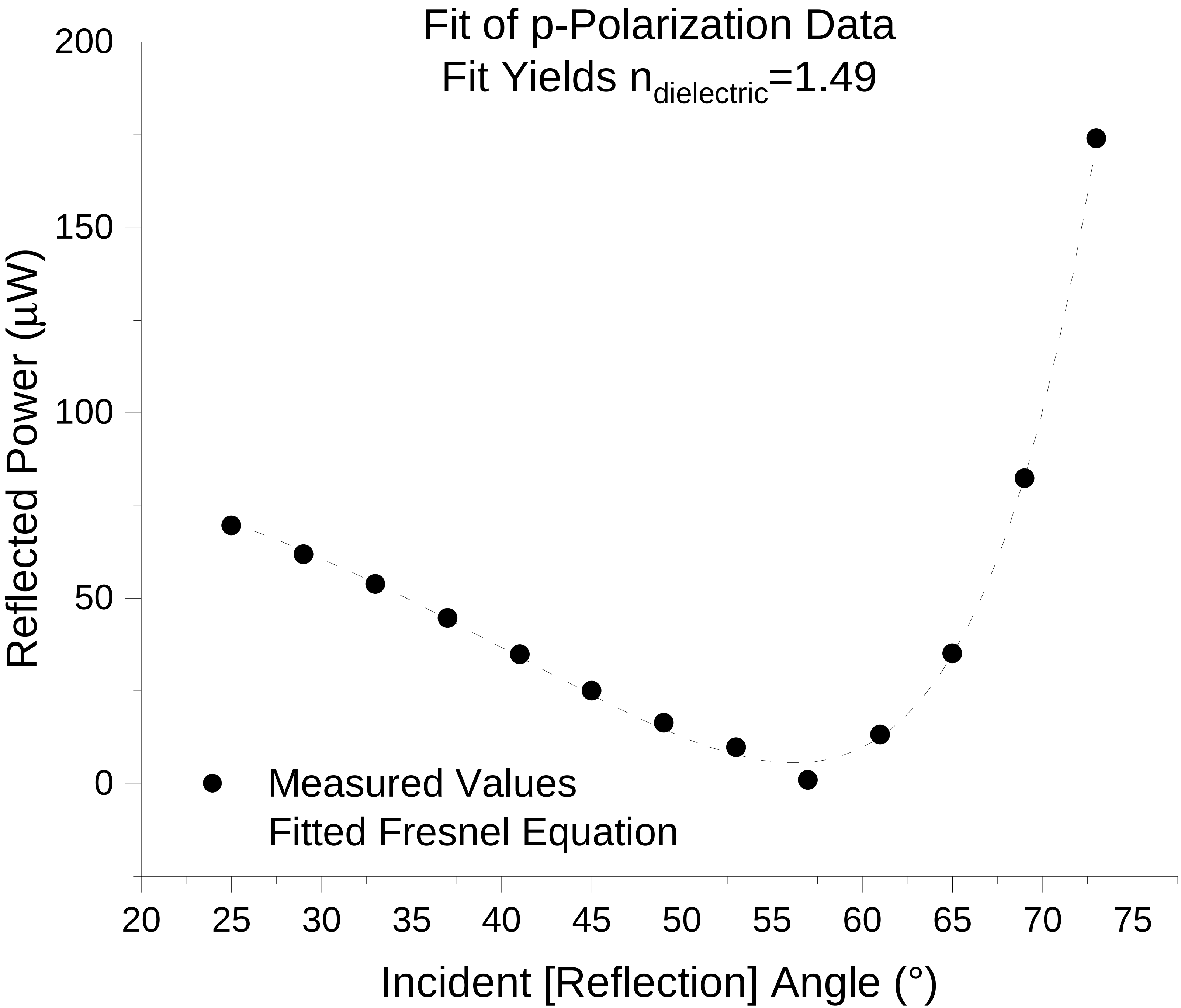

Figure 1: A representation of the relevant geometry for expressing the physics of the interaction of a plane electromagnetic wave at a dielectric boundary. Note that the representation does ***not*** include a transmitted component of the wave which, except in very specific circumstances, would also be present.

- the vectors $\hat{\imath}$ and $\hat{r}$ represent the direction of propagation of the incident and reflected plane electromagnetic wave, respectively
- the vector $\hat{n}$ is a unit vector normal (perpendicular) to the plane of the dielectric boundary
- the ***Plane of Incidence*** is defined as a plane that contains the three vectors $\hat{\imath}$ , $\hat{r}$, and $\hat{n}$ ; NOTE that it is the geometry of those vectors that determine the plane of incidence and it is essential to understand that you ***cannot***, for instance, think of the plane of incidence as generally horizontal or vertical – it depends on the geometry of the direction of propagation of the plane electromagnetic wave and the orientation of the dielectric boundary
- the angles $\theta_i$ and $\theta_r$ are the angles of incidence and reflection; NOTE that they are defined relative to $\hat{n}$ , the normal to the dielectric boundary; for specular reflection, the "law of reflection" applies and $\theta_r = \theta_i$
- the direction of the electric field vector for a plane electromagnetic wave must be perpendicular to the direction of propagation of the wave
- the relevant physics for fully expressing the behavior of the plane electromagnetic wave at the boundary requires resolving the the electric field of the wave into two specific components (remember that both of these components are still perpendicular to the direction of propagation of the wave, and that the electric field at a location is changing in time in accordance with the specific polarizaton of the wave)
  - the *s* component (mnemonic to remember: ***s***ticking out) is the component of the electric field of the plane electromagnetic wave that is perpendicular to the plane of incidence (and also to the direction of propagation of the wave)
  - the *p* component (mnemonic to remember: in the ***p***lane) is the component of the electric field of the plane electromagnetic wave that lies in the plane of incidence (and is perpendicular to the direction of propagation of the wave)
  - the *s* and *p* components of the electric field of a plane electromagnetic wave will be sinusoidally oscillatory, each component with its own constant amplitude; the specific polarization state of the wave determines the relative amplitude and phase of the components
  - the *s* and *p* components of both the incident and reflected plane electromagnetic wave are represented in Fig. 1
  - the behavior of these two components of the wave as a result of the interaction at the dielectric boundary ***differs*** and so it is essential to be able to distinguish them in fully representing the interaction

Fresnel Equations Student Tasks List

Following the completion of each task, verify and/or discuss your work with the instructor ***before*** proceeding to the next task.

1. Using alignment tool and mirrors on kinematic mounts, align the laser beam to be parallel to the table and so that it contacts the dielectric surface at the vertical axis of rotation of the surface.
2. Develop a scheme to calibrate the angle scale of the rotation stage on which the dielectric is mounted. The calibration scheme needs to permit a reading of the angle on the scale of the dielectric rotation stage to be used to obtain the angle of incidence of the laser beam on the dielectric.
3. Using the screen [index card] on the swing arm, ***qualitatively*** investigate the phenomena associated with the light ***reflected*** from the dielectric surface [ignore the transmitted beam and its multiple reflections, at least for now]. In your qualitative investigation, the only two parameters you should vary are the angle of incidence [using the rotation stage of the dielectric], and the angle of the half-wave plate [using its rotation mount]. Make sure to note any interesting behavior observed.
4. Develop a calibration scheme for the angle scale of the half-wave plate rotation mount. The calibration scheme needs to permit a reading of the angle on the scale of the half-wave plate rotation mount to be used to obtain the angle of the linear polarization of the laser beam relative to the p-polarization for the system of laser beam and dielectric.
5. Attach the laser power meter to the swing arm and perform the calibration from step 4 above using the meter.
6. Measure the relationships corresponding the to Fresnel equation for p- and s- polarization for the system.
7. Fit the data collected in step 6 above using the Fresnel equations as a model. An essential finding from the fit is the value of the index of refraction for the dielectric.


Fit of p-Polarization Data
Fit Yields $n_{dielectric}$=1.49
Reflected Power (μW)
200
150
100
50
0
Measured Values
Fitted Fresnel Equation
20
25
30
35
40
45
50
55
60
65
70
75
Incident [Reflection] Angle (°)

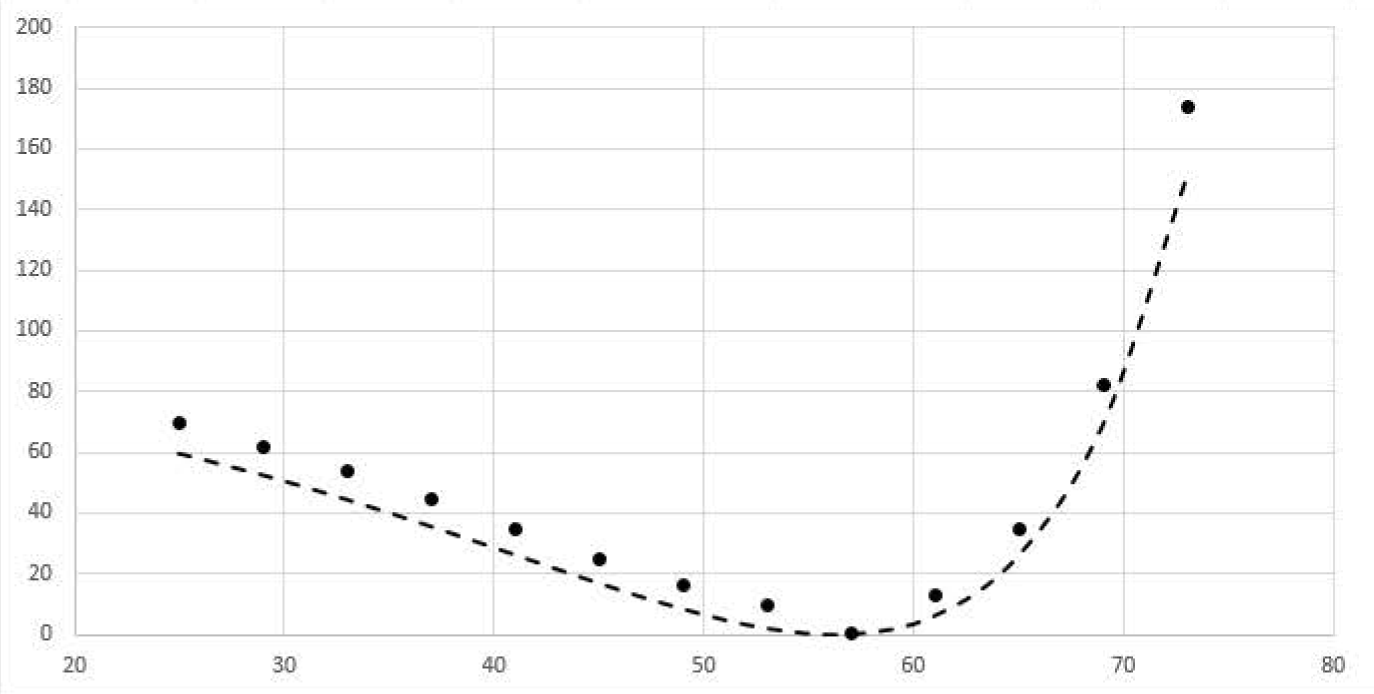
fresnel fit in excel.xlsx - Excel

File | Home | Insert | Page Layout | Formulas | Data | Review | View | Developer | Add-ins | Acrobat | Tell me what you want to do... | Michael Braunstein | Sha

Get External Data: From Access, From Web, From Text, From Other Sources, Existing Connections
Get & Transform: New Query, Show Queries, From Table, Recent Sources
Connections: Refresh All, Connections, Properties, Edit Links
Sort & Filter: Sort, Filter, Clear, Reapply, Advanced
Data Tools: Text to Columns, Flash Fill, Remove Duplicates, Data Validation, Consolidate, Relationships, Manage Data Model
Forecast: What-If Analysis, Forecast Sheet
Outline: Group, Ungroup, Subtotal, Show Detail, Hide Detail
Analyze: Data Analysis, Solver

L19 | fx

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | measured theta | measured I | | theta radians | t1 | s1 | t2 | num | den | Fresnel Model | | delta sqr |
| 2 | | | | | | | | | | | | |
| 3 | 25 | 69.6 | | 0.436332313 | 2.039192521 | 0.422618 | 1.439234 | 0.599959 | 3.478426 | 59.49866122 | | 102.037 |
| 4 | 29 | 61.8 | | 0.506145483 | 1.967894341 | 0.48481 | 1.419493 | 0.548402 | 3.387387 | 52.42009147 | | 87.98268 |
| 5 | 33 | 53.8 | | 0.575958653 | 1.887008778 | 0.544639 | 1.39763 | 0.489379 | 3.284638 | 44.3962178 | | 88.43112 |
| 6 | 37 | 44.7 | | 0.645771823 | 1.796929898 | 0.601815 | 1.373979 | 0.422951 | 3.170909 | 35.5829575 | | 83.12046 |
| 7 | 41 | 34.8 | | 0.715584993 | 1.698096556 | 0.656059 | 1.348921 | 0.349176 | 3.047017 | 26.26449764 | | 72.8548 |
| 8 | 45 | 25.1 | | 0.785398163 | 1.590990258 | 0.707107 | 1.322876 | 0.268115 | 2.913866 | 16.93291796 | | 66.70123 |
| 9 | 49 | 16.4 | | 0.855211333 | 1.476132815 | 0.75471 | 1.296308 | 0.179825 | 2.77244 | 8.41407968 | | 63.77492 |
| 10 | 53 | 9.7 | | 0.925024504 | 1.354083802 | 0.798636 | 1.269717 | 0.084367 | 2.623801 | 2.067812662 | | 58.25028 |
| 11 | 57 | 1 | | 0.994837674 | 1.225437829 | 0.838671 | 1.243636 | -0.0182 | 2.469074 | 0.108652513 | | 0.7945 |
| 12 | 61 | 13.2 | | 1.064650844 | 1.090821646 | 0.87462 | 1.218622 | -0.1278 | 2.309444 | 6.124652494 | | 50.06054 |
| 13 | 65 | 35.1 | | 1.134464014 | 0.950891089 | 0.906308 | 1.195243 | -0.24435 | 2.146134 | 25.92676029 | | 84.14833 |
| 14 | 69 | 82.4 | | 1.204277184 | 0.806327886 | 0.93358 | 1.174065 | -0.36774 | 1.980392 | 68.96065162 | | 180.6161 |
| 15 | 73 | 174 | | 1.274090354 | 0.657836336 | 0.956305 | 1.15563 | -0.49779 | 1.813467 | 150.6988758 | | 542.9424 |
| 18 | | | params | | | | | | | | | |
| 19 | | | I0 | 2000 | | | | | | | sum delta sqrs | 1481.714 |
| 20 | | | n | 1.5 | | | | | | | | |
| 21 | | | c0 | 0 | | | | | | | | |




**Solver Parameters**

Set Objective: $L$19

To: Max / Min (selected) / Value Of: 0

By Changing Variable Cells: $D$19:$D$21

Subject to the Constraints:

Add | Change | Delete | Reset All | Load/Save

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear | Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help | Solve | Close

Sheet1

Point