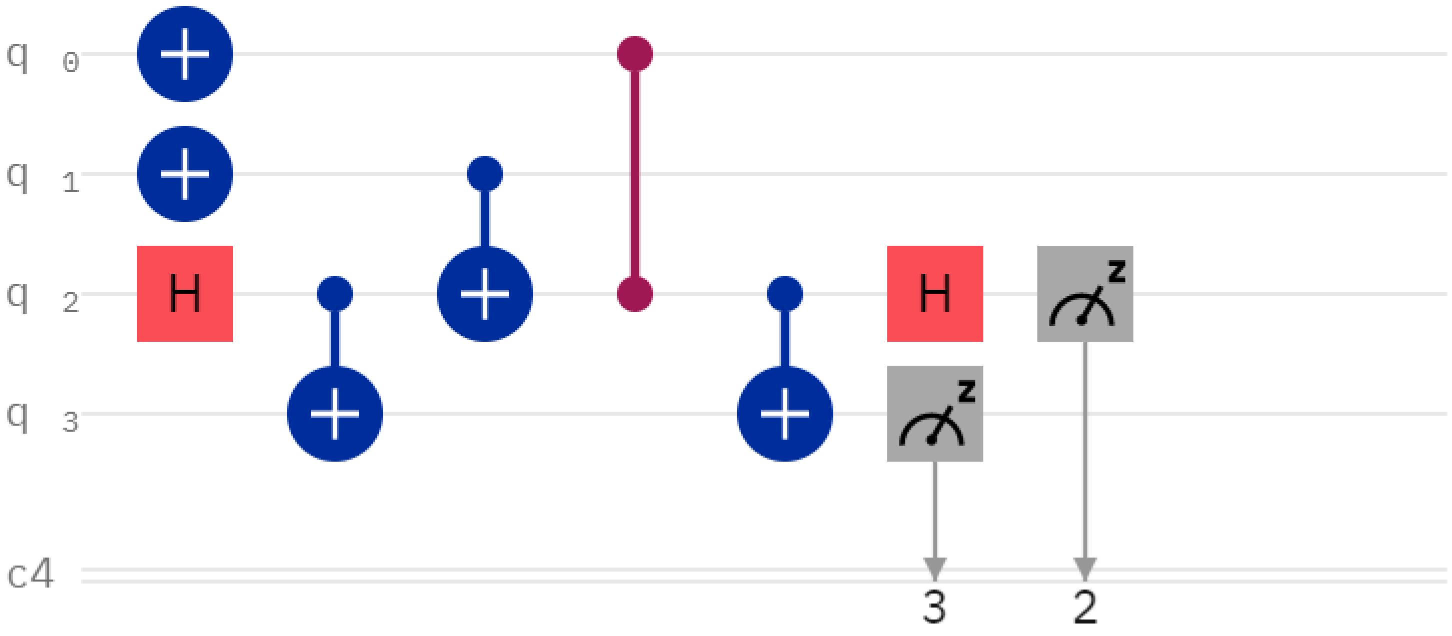
Quantum Computing with IBM Quantum



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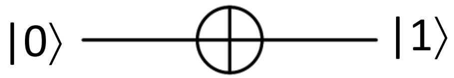
Fundamentals

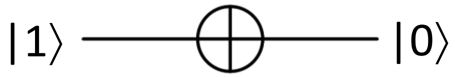
- A measurement of a qubit in the "computational basis" yields 0 or 1
- Prior to measurement, a qubit may be in a superposition, $\alpha|0\rangle + \beta|1\rangle$:
  - $|\alpha|^2 + |\beta|^2 = 1$
  - $|\alpha|^2$ is the probability of measuring 0
  - $|\beta|^2$ is the probability of measuring 1
- Unitary operators called "gates" manipulate qubits to change the amplitudes (coefficients) of $|0\rangle$ and $|1\rangle$

Example: NOT gate and Z gate

- $X|0\rangle = |1\rangle$

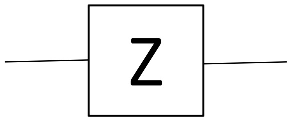


- $X|1\rangle = |0\rangle$



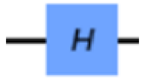
- $X(\alpha|0\rangle + \beta|1\rangle) = \alpha X|0\rangle + \beta X|1\rangle = \alpha|1\rangle + \beta|0\rangle$

- $Z|0\rangle = |0\rangle$
- $Z|1\rangle = -|1\rangle$



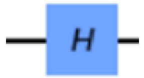
Example: Hadamard gate

- $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|0\rangle$ 

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

- $H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

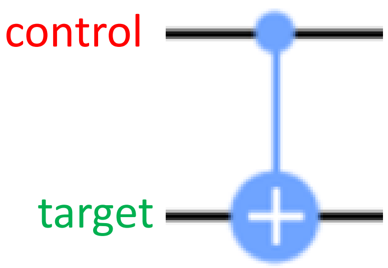
$|1\rangle$ 

$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

- $H(\alpha|0\rangle + \beta|1\rangle) = \alpha H|0\rangle + \beta H|1\rangle = \frac{\alpha}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{\beta}{\sqrt{2}}(|0\rangle - |1\rangle)$

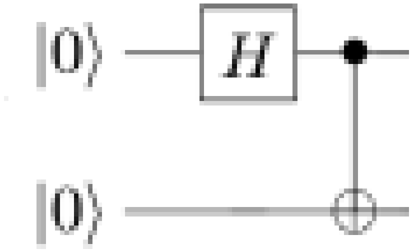
$$= \frac{1}{\sqrt{2}}[(\alpha + \beta)|0\rangle + (\alpha - \beta)|1\rangle]$$

Controlled NOT



- Almost every textbook writes **top** qubit **first** (I follow this convention)
- IBM Quantum writes top qubit last (very unconventional)
- CNOT$|0\rangle|0\rangle = |0\rangle|0\rangle$
- CNOT$|0\rangle|1\rangle = |0\rangle|1\rangle$
- CNOT$|1\rangle|0\rangle = |1\rangle|1\rangle$
- CNOT$|1\rangle|1\rangle = |1\rangle|0\rangle$
- CNOT$|x\rangle|y\rangle = |x\rangle|y\oplus x\rangle$
- CNOT$(\alpha|0\rangle|0\rangle+\beta|0\rangle|1\rangle+\gamma|1\rangle|0\rangle+\delta|1\rangle|1\rangle)=\alpha|0\rangle|0\rangle+\beta|0\rangle|1\rangle+\gamma|1\rangle|1\rangle+\delta|1\rangle|0\rangle$

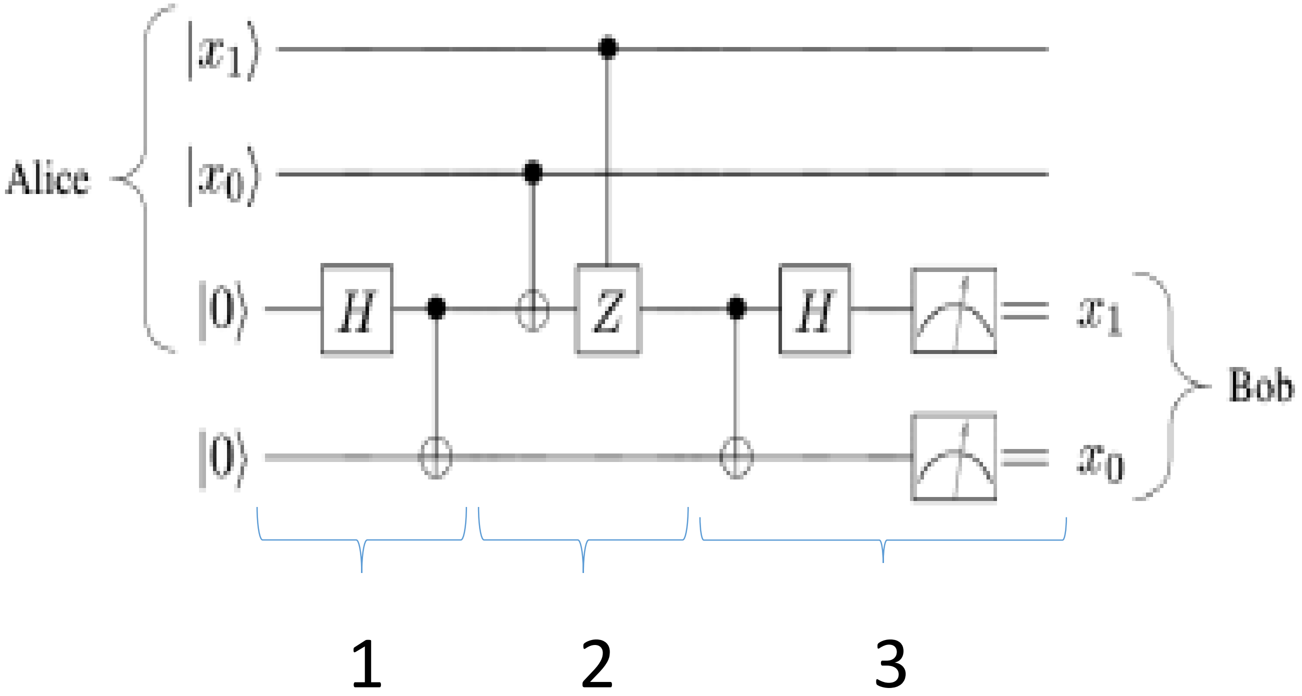
Quantum Entanglement



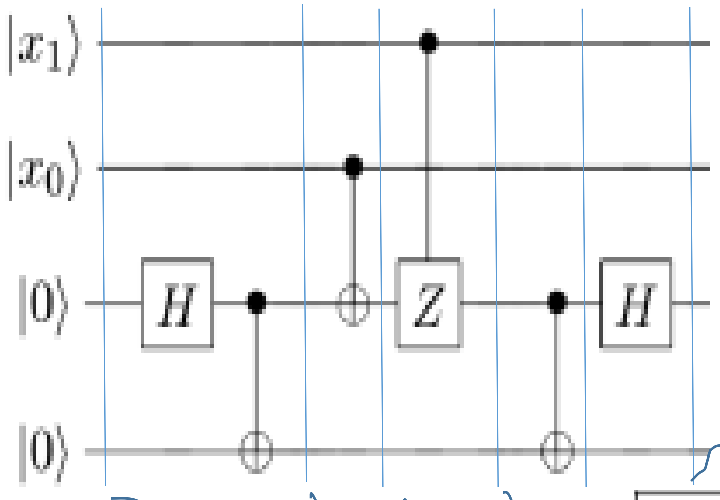
- Initial state: $|0\rangle|0\rangle$
- After the H: $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|0\rangle)$
- After the CNOT: $\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$

Quantum Dense Coding

(figure from Garcia-Martin and Sierra, arxiv.org/pdf/1712.05642.pdf)



1. Alice and Bob meet to create a pair of entangled qubits
2. Alice departs and encodes two bits, x_1 and x_0, into third qubit from top
3. Alice sends this one qubit (encoding two bits) to Bob to decode



Proof

- From lab report by Howard Hu
- Alternatively, leave x_1 and x_0 symbolic
- Experiment: https://quantum-computing.ibm.com/

Initial	$\|0000\rangle$	$\|0100\rangle$	$\|1000\rangle$	$\|1100\rangle$
1^{st} + 2^{nd} states	$\frac{1}{\sqrt{2}}(\|0000\rangle + \|0011\rangle)$	$\frac{1}{\sqrt{2}}(\|0100\rangle + \|0111\rangle)$	$\frac{1}{\sqrt{2}}(\|1000\rangle + \|1011\rangle)$	$\frac{1}{\sqrt{2}}(\|1100\rangle + \|1111\rangle)$
3^{rd} state	$\frac{1}{\sqrt{2}}(\|0000\rangle + \|0011\rangle)$	$\frac{1}{\sqrt{2}}(\|0110\rangle + \|0101\rangle)$	$\frac{1}{\sqrt{2}}(\|1000\rangle + \|1011\rangle)$	$\frac{1}{\sqrt{2}}(\|1110\rangle + \|1101\rangle)$
4^{th} state	$\frac{1}{\sqrt{2}}(\|0000\rangle + \|0011\rangle)$	$\frac{1}{\sqrt{2}}(\|0110\rangle + \|0101\rangle)$	$\frac{1}{\sqrt{2}}(\|1000\rangle - \|1011\rangle)$	$\frac{1}{\sqrt{2}}(-\|1110\rangle + \|1101\rangle)$
5^{th} state	$\frac{1}{\sqrt{2}}(\|0000\rangle + \|0010\rangle)$	$\frac{1}{\sqrt{2}}(\|0111\rangle + \|0101\rangle)$	$\frac{1}{\sqrt{2}}(\|1000\rangle - \|1011\rangle)$	$\frac{1}{\sqrt{2}}(-\|1111\rangle + \|1101\rangle)$
6^{th} state	$\|0000\rangle$	$\|0101\rangle$	$\|1010\rangle$	$\|1111\rangle$

Deutsch's problem

- Consider a circuit with a single-bit input x and a single-bit output f(x)
- There are 4 possible functions (truth tables):

x	f(x)
0	0
1	0

x	f(x)
0	0
1	1

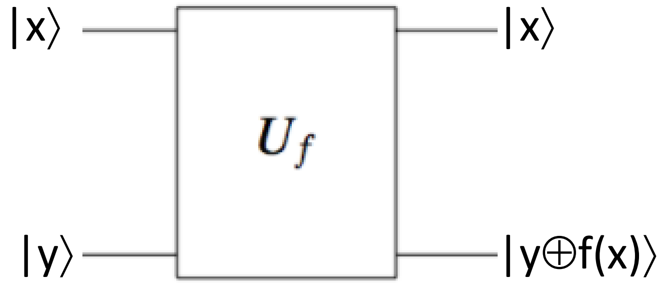
x	f(x)
0	1
1	0

x	f(x)
0	1
1	1

- How to determine experimentally if f(x) is a **constant** (0 or 1)?
- In classical (non-quantum) circuit, we'd have to take the time to input both possible values of x, one after the other
- Deutsch's solution uses just a **single** application of quantum circuit

Oracle operator: quantum gate incorporating f(x)

- Define oracle operator $U_f|x\rangle|y\rangle=|x\rangle|y\oplus f(x)\rangle$



- All four oracles (maybe give students two):
  - If $f(x)=0$, $U_f|x\rangle|y\rangle=|x\rangle|y\oplus 0\rangle=|x\rangle|y\rangle$; U_f is nothing
  - If $f(x)=1$, $U_f|x\rangle|y\rangle=|x\rangle|y\oplus 1\rangle=|x\rangle|\bar{y}\rangle$; U_f is NOT on bottom qubit
  - If $f(x)=x$, $U_f|x\rangle|y\rangle=|x\rangle|y\oplus x\rangle=\text{CNOT}|x\rangle|y\rangle$; U_f is CNOT
  - If $f(x)=\bar{x}$, $U_f|x\rangle|y\rangle= |x\rangle|y\oplus\bar{x}\rangle= |x\rangle|\bar{y}\oplus x\rangle=\text{CNOT}|x\rangle|\bar{y}\rangle$; U_f is NOT and CNOT, order flexible

Deutsch's Algorithm

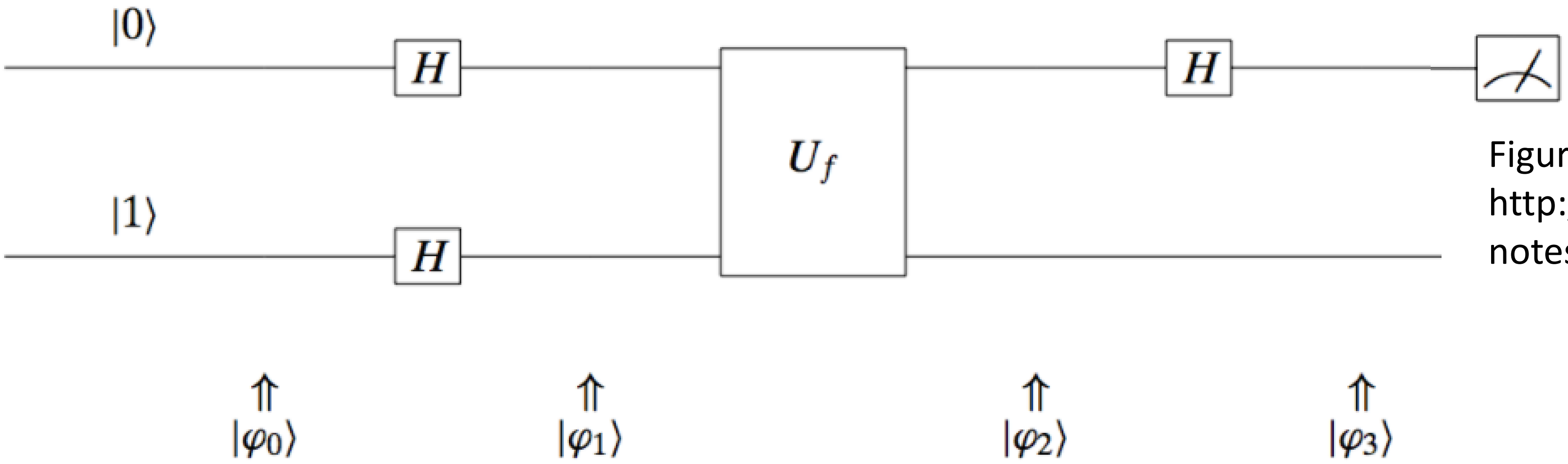


Figure from http://akyrillidis.github.io/notes/quant_post_8

- $|\phi_0\rangle = |0\rangle|1\rangle$
- $|\phi_1\rangle = H|0\rangle H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{2}(|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle)$
- $|\phi_2\rangle = U_f|\phi_1\rangle = \frac{1}{2}(|0\rangle|f(0)\rangle - |0\rangle|1 \oplus f(0)\rangle + |1\rangle|f(1)\rangle - |1\rangle|1 \oplus f(1)\rangle)$
- $|\phi_3\rangle = \frac{1}{2}(H|0\rangle|f(0)\rangle - H|0\rangle|1 \oplus f(0)\rangle + H|1\rangle|f(1)\rangle - H|1\rangle|1 \oplus f(1)\rangle)$

Measure top qubit to see if f(x) is constant

- If f(x) is constant: f(0)=f(1) $\Rightarrow$

$|\phi_3\rangle = \frac{1}{2}(H|0\rangle|f(0)\rangle - H|0\rangle|1\oplus f(0)\rangle + H|1\rangle|f(0)\rangle - H|1\rangle|1\oplus f(0)\rangle)$

$= \frac{1}{2}(H|0\rangle + H|1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$

$= \frac{1}{\sqrt{2}}|0\rangle(|f(0)\rangle - |1\oplus f(0)\rangle) \Rightarrow$ **top qubit is $|0\rangle$**

- If f(x) is not constant: f(1)=1⊕f(0) $\Rightarrow$

$|\phi_3\rangle = \frac{1}{2}(H|0\rangle|f(0)\rangle - H|0\rangle|1\oplus f(0)\rangle + H|1\rangle|1\oplus f(0)\rangle - H|1\rangle|f(0)\rangle)$

$= \frac{1}{2}(H|0\rangle - H|1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$

$= \frac{1}{\sqrt{2}}|1\rangle(|f(0)\rangle - |1\oplus f(0)\rangle) \Rightarrow$ **top qubit is $|1\rangle$**

Resources

- https://quantum-computing.ibm.com/
- Qiskit textbook: https://qiskit.org/textbook/preface.html
- My favorite quantum computing textbooks:
  - De Lima Marquezino *et al*. (2019), *A Primer on Quantum Computing*
  - Zygelman (2018), *A First Introduction to Quantum Computing*
- Garcia-Martin and Sierra, "Five Experimental Tests on the 5-Qubit IBM Quantum Computer": https://arxiv.org/abs/1712.05642
- Recent AJP papers:
  - Kain, Searching a quantum database with Grover's search algorithm, 2021: https://aapt.scitation.org/doi/abs/10.1119/10.0004835
  - Brody and Guzman, Calculating spin correlations with a quantum computer, 2020: https://aapt.scitation.org/doi/abs/10.1119/10.0001967
- Please feel free to contact me: jbrody@emory.edu. I'm not an expert, but I'd be delighted to hear from you!